

# Self-similarity of Dynamo Action in the Largest Cosmic Structures

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Massive galaxy clusters (GC) are filled with a hot, turbulent and magnetised intra-cluster medium (ICM). Still forming under the action of gravitational instability they grow in mass by accretion of supersonic flows. These flows partially dissipate into heat through a complex network of large scale shocks<sup>1</sup>, while residual transonic flows create giant turbulent eddies and cascade<sup>2,3</sup>. Turbulence heats the ICM<sup>4</sup> and also amplifies magnetic energy by way of dynamo action<sup>5-8</sup>. However, fundamental properties of the pattern whereby gravitational energy turns kinetic, thermal, turbulent and magnetic remain unknown. Here we find that the energy components of the ICM are ordered according to a permanent hierarchy, in which the ratios of thermal to turbulent to magnetic energy densities remain virtually unaltered throughout the ICM history despite evolution of each individual component and the drive towards equipartition of turbulent dynamo. Our results are based on a state-of-the-art, fully cosmological computational model of ICM turbulence<sup>3,9</sup>, revealing that an approximately constant efficiency of turbulence generation from gravitational energy that is freed during mass accretion. The permanent character of this hierarchy reflects a new aspect of self-

**similarity in cosmology<sup>10–13</sup>, while its structure, consistent with current data<sup>14–18</sup>, encodes information about the efficiency of turbulent heating and dynamo action.**

The computational model captures the turbulent motions through a multi scale technique which employs six nested grids covering progressively larger volumes with correspondingly coarser resolution elements<sup>3,19</sup>. The finest grid resolves the virial volume of the GC with more than a billion uniform-size resolution elements and provide the necessary dynamic range to resolve the ICM turbulent cascade. The largest grid covers the chosen cosmological volume of 340 comoving Mpc on a side (comoving=partaking in the expansion of the universe; 1 Mpc  $\approx$  3 million ly). The intermediate grids allow to simultaneously follow with adequate accuracy the matter distribution outside the GC volume. The calculation starts with three grids and adds progressively finer grids as the Lagrangian volume of the GC shrinks under self-gravity. All six grids are in place at a time corresponding to 8 billion yr after Big Bang. At current time (13.8 billion years after Big Bang) the simulated GC has a total virial mass of  $1.3 \times 10^{15} M_{\odot}$ .

Figure 1 shows a snapshot of the simulation illustrating the cosmological context and the highly turbulent conditions of the flow inside the GC volume. The exquisite resolution across the GC volume allows us to accurately measure the time dependent statistical properties of structure formation driven ICM turbulence including, in particular, the dissipation rate,  $\epsilon_{turb}$ , the outer scale,  $L$  and the velocity dispersion on that scale,  $\langle (\delta u_L)^2 \rangle^{\frac{1}{2}}$  (Methods). In the following we restrict our analysis to a region within 1/3 of the GC’s virial radius,  $R_{vir}$ , where  $R_{vir}$  defines a region characterised by a mass over-density  $\Delta_c \approx 100$  that has nominally reached dynamical equilibrium.

Our choice is motivated by the fact that at current epoch  $R_{vir}/3 \approx 1$  Mpc, i.e. defines a region most relevant for comparison with observations.

Hydrodynamic turbulence is dominated by the solenoidal component accounting for 60-90% of the total kinetic energy<sup>9,20</sup>. Detailed analysis shows that this component remains statistically homogenous and isotropic thus resembling Kolmogorov's cascade, despite the presence of considerable structure in the ICM<sup>9</sup>. The dissipation of incompressible turbulence contributes to ICM heating along with shocks and adiabatic compression and to the growth of magnetic energy by way of small scale dynamo action<sup>5,8</sup> (Methods: Fig. 4 for cascade details). The turbulent dissipation rate associated to the solenoidal component is estimated from the numerical simulation data. Because ICM turbulence is driven by various complex hydrodynamic mechanisms ultimately powered by the unsteady mass accretion process<sup>9</sup>, the dissipation rate is highly changeable with time and exhibits non-monotonic variations by more than one order of magnitude<sup>20</sup> (Figure 2c). However, alongside with much complexity turbulent dissipation appears to also exhibit simplicity of behaviour. This is shown in Figure 2a illustrating the time evolution of the fraction of thermal energy originating from turbulent dissipation. In contrast to  $\epsilon_{turb}$ , this quantity remains remarkably constant during the GC lifetime,  $\eta_{turb} \approx 0.3 - 0.4$ , indicating that the efficiency of turbulence generation out of gravitational energy freed by mass accretion is approximately constant. In addition, Figure 2b shows that the turbulence velocity dispersion at the outer scale normalised to the ICM sound speed, i.e. the turbulence Mach number  $\mathcal{M}_{turb} = \langle (\delta u_L)^2 \rangle^{1/2} / c_s$ , also remains rather constant with time. This shows that in the ICM the evolution of the turbulent kinetic energy and the thermal energy are closely related, consistent with the previous plot. The value of  $\mathcal{M}_{turb}$  can be under-

stood as follows. If the generation of the bulk of the thermal energy,  $E_{th}$ , is dominated by the last  $\alpha = 2 - 3$  eddy turnover times,  $\tau_L = L/u_L$ , then  $E_{th} \simeq \eta_{turb}^{-1} \int \rho \epsilon_{turb} dt \simeq (\alpha/3^{\frac{3}{2}} \eta_{turb}) \rho \langle (\delta u_L)^2 \rangle$ , where we have used the known relation  $\epsilon_{turb} = (2/3C)^{\frac{3}{2}} \langle (\delta u_L)^2 \rangle^{\frac{3}{2}} / L$  with  $C \approx 2^{21}$ . It is straightforward to then see that the Mach number  $\mathcal{M}_{turb} \approx (\sqrt{3}/\alpha)^{\frac{1}{2}} (\eta_{turb}/0.37)^{\frac{1}{2}}$  which, for  $\alpha = 1.5 - 3$  ranges between 0.8 and 1.2 confirming the result in Figure 2. It also follows that the ratio of thermal to turbulent kinetic energy is

$$\frac{E_{th}}{\frac{1}{2} \rho \langle (\delta u_L)^2 \rangle} \approx \frac{2\alpha}{3^{\frac{3}{2}}} \eta_{turb}^{-1} \approx \eta_{turb}^{-1}. \quad (1)$$

Generation of magnetic field by small scale dynamo in a turbulent flow follows from standard theory. In a high Reynolds number ( $Re > 10^3$ ) flow such as the ICM an initial seed of vanishing strength<sup>22-27</sup> is amplified exponentially at the rate  $\gamma = \sqrt{Re}/30\tau_L$ , where  $\tau_L = L/\delta u_L$  is the eddy turnover time. After a short while ( $\propto Re^{-1/2}$ ) magnetic field stops growing below a characteristic Alfvén scale,  $L_A \equiv v_A^3 / C^{\frac{3}{2}} \epsilon_{turb}$ , where  $v_A = B/\sqrt{4\pi\rho}$  is the Alfvén speed, due to the feedback action of magnetic tension<sup>5,8</sup>. Magnetic energy continues to grow at the expenses of turbulent kinetic energy as  $L_A$ , marking the equipartition scale between kinetic and magnetic energy, shifts towards larger values<sup>5</sup>. Growth, however, is now proportional to the turbulent dissipation rate instead of exponential with time. It is in this latter stage that the dynamo spends most of the time<sup>8</sup>. Recent state-of-the-art numerical work finds that for statistically isotropic and homogeneous turbulence, as found in the ICM<sup>9,20</sup>, the efficiency of conversion of turbulent (kinetic) to magnetic energy is a universal number ca  $C_E = 4-5\%$ <sup>8</sup>.

Therefore, the evolution of magnetic energy in the ICM can be expressed in terms of the tur-

bulence dissipation history as  $E_B(t) = B^2/8\pi = C_E \int^t d\tau \rho \epsilon(\tau)$ . Combined with the above finding about  $\eta_{turb}$ , this leads to simple but significant expressions relating the fundamental properties of magnetic field and turbulence in the ICM. In fact, since turbulence dissipation contributes a constant fraction,  $\eta_{turb} \approx 1/3$ , of ICM thermal energy, the ratio  $\beta_{plasma}$  of thermal pressure to magnetic energy can be written as

$$\beta_{plasma} \equiv \frac{P_{gas}}{B^2/8\pi} = \frac{\eta_{turb}^{-1}(\gamma - 1)}{C_E} = 40 \left( \frac{\eta_{turb}}{1/3} \right)^{-1} \left( \frac{C_E}{0.05} \right)^{-1}. \quad (2)$$

This means that for massive GC  $\beta_{plasma}$  is a constant, which depends neither on the specifics of the ICM conditions including turbulence, nor on the GC mass or age. It is instead simply determined by two fundamental parameters,  $C_E$  and  $\eta_{turb}$ , which describe the efficiency of turbulent dynamo and of turbulent heating in structure formation, respectively. This is shown in Figure 3a where  $\beta_{plasma}$  is plotted as a function of cosmic time and exhibits 25% rms fluctuations, which should also characterise massive cluster-to-cluster variations. We can also compute the Alfvén scale. Since the turbulence is non-stationary and the magnetic energy retains memory over more than one eddy turnover time, we average the dissipation rate  $\epsilon_{turb}$  over 2 Gyr when calculating  $L_A$ . Expressing  $L_A$  in units of the turbulence outer scale we write

$$\frac{L_A}{L} \equiv \frac{v_A^3}{C^{\frac{3}{2}} \langle \epsilon_{turb} \rangle} = \frac{3}{2} \left( \frac{2}{\gamma \beta_{plasma}} \right)^{\frac{3}{2}} \frac{c_s^3}{\langle (\delta u_L)^2 \rangle^{\frac{3}{2}}} = \frac{1}{100} \left( \frac{\beta_{plasma}}{40} \right)^{-\frac{3}{2}} \left( \frac{\mathcal{M}_{turb}}{1} \right)^{-3}. \quad (3)$$

We have already shown that both  $\beta_{plasma}$  and the turbulence Mach number,  $\mathcal{M}_{turb} = \langle (\delta u_L)^2 \rangle^{\frac{1}{2}} / c_s$ , remain constant during the evolution of the GC. Therefore, the Alfvén scale too remains a constant fraction of the turbulence driving scale, independent of time, GC mass and ICM conditions. In addition, given the large value of  $\beta_{plasma}$ , and that  $\mathcal{M}_{turb} \approx 1$ ,  $L_A$  is small compared to  $L$ . The

time evolution of  $L_A/L$  is shown in Figure 3b (see Fig. 5 in Methods for a typical ICM spectrum of hydromagnetic turbulence). Finally, Figure 3c shows that the evolution of the turbulent injection scale,  $L$ , closely follows that of  $L_A$  while tracking the growing characteristic scale of the GC ( $R_{500} = 0.5R_{vir}$ ) also plotted in the same panel. Note that the modulation of  $L_A$  reflects the changing turbulent conditions in the ICM and, in particular, is anti correlated with  $\epsilon_{turb}$ , as generally expected.

The large value of  $\beta_{plasma}$  indicates that magnetic energy, like turbulent energy, is small compared to thermal energy ( $E_{th} \gg E_B$ ). Moreover, the small value of  $L_A/L$  indicates that the dynamo is far from saturation and magnetic energy is also small in comparison to the turbulent kinetic energy ( $E_{turb} \gg E_B$ ). This energy hierarchy is fundamentally due to the efficiency  $\eta_{turb}$  with which turbulent energy is generated during gravitational collapse and the fraction  $C_E$  thereof that is converted into magnetic energy, namely  $E_{th} : E_{turb} : E_B = 1 : \eta_{turb} : C_E \eta_{turb}$ . The values of  $\beta_{plasma}$  and  $L_A$  are in good agreement with recent measurements of magnetic field properties in GC <sup>14–18</sup>. Here, they emerge from pure numerical modelling of structure formation turbulence and MHD dynamo action, in the sense that they are found to derive their values from the parameters  $\eta_{turb}$  and  $C_E$ , which are determined numerically and not through parametric fits. Intriguingly, the above energy hierarchy appears to remain unchanged during the GC evolution and the turbulent dynamo in the ICM is far from saturation today as it has virtually always been in the past. Figure 3c shows that the GC size and, therefore, its mass constantly grow. This implies that the gravitational potential energy and therefore the ICM thermal energy and turbulent energy also continue to grow. Meanwhile dynamo action tries to bring magnetic and turbulent energy into equipartition. Since

all of these forms of energy grow simultaneously but with different constant efficiencies, their ratio remains unchanged, reflecting the value of those intrinsic efficiencies. In other words, both  $\beta_{plasma}$  and  $L_A/L$  encode the efficiency of turbulent generation in structure formation and the efficiency of dynamo action. As such, they allow us to relate magnetic field observations in massive galaxy clusters to such properties of structure formation. This is in sharp contrast with other astrophysical bodies<sup>28–30</sup>, e.g. the interstellar medium of galaxies, stars and compact objects, where the turbulence dynamo has long saturated and such information is lost forever.

## Methods

**Numerical model** The simulation is carried out with CHARM, an Adaptive-Mesh-Refinement cosmological code<sup>19</sup>. This code uses a directionally un-split variant of the piecewise parabolic method for hydrodynamics<sup>31</sup>, constrained-transport algorithm for solenoidal MHD<sup>32</sup>, a time-centred modified symplectic scheme for the collision-less dark matter, and solve Poissons equation with a second-order accurate discretisation. The magnetic field remains negligible throughout, so the calculation is effectively hydrodynamic. For massive galaxy clusters, such as Coma cluster, the ICM cooling time is a few times the age of the universe<sup>33</sup>, so cooling and baryonic feedback processes are neglected. Heating of the intergalactic medium through photoionization is also neglected, with no consequences whatsoever for the generation of vorticity and turbulence at accretion shocks. We use a concordance  $\Lambda$ -CDM universe with normalized (in units of the critical value) total mass density,  $\Omega_m = 0.2792$ , baryonic mass density,  $\Omega_b = 0.0462$ , vacuum energy density,  $\Omega_\Lambda = 1 - \Omega_m = 0.7208$ , normalized Hubble constant  $h \equiv H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1} = 0.701$ , spectral index of primordial perturbation,  $n_s = 0.96$ , and rms linear density fluctuation within a sphere with a comoving radius of  $8 h^{-1} \text{ Mpc}$ ,  $\sigma_8 = 0.817$ <sup>34</sup>. The simulated volume has comoving size of  $L_{Box} = 240 h^{-1} \text{ Mpc}$  on a side. The initial conditions are generated on three refinement levels with `grafic++` (made publicly available by D. Potter). For the coarsest level we use  $512^3$  comoving cells, corresponding to a nominal spatial resolution of  $468.75 h^{-1} \text{ comoving kpc}$  and  $512^3$  particles of mass  $6.7 \times 10^9 h^{-1} M_\odot$  to represent the collisionless dark matter component. The additional levels allow for refined initial conditions in the volume where the galaxy cluster forms. The refinement ratio for both levels is,  $n_{\text{ref}}^\ell \equiv \Delta x_\ell / \Delta x_{\ell+1} = 2$ ,  $\ell = 0, 1$ . Each refined level covers



1/8 of the volume of the next coarser level with a uniform grid of  $512^3$  comoving cells while the dark matter is represented with  $512^3$  particles. At the finest level the spatial resolution is  $\Delta x = 117.2 h^{-1}$  comoving kpc and the particle mass is  $10^8 h^{-1} M_\odot$ . As the Lagrangian volume of the galaxy cluster shrinks under self-gravity, three additional uniform grids covering 1/8 of the volume of the next coarser level are employed with  $512^3$ ,  $1,024^3$  and  $1,024^3$  comoving cells, respectively, and  $n_{\text{ref}}^\ell = 2, 4, 2$ , for  $\ell = 2, 3, 4$ , respectively. All of them are in place by redshift 1.4, providing a spatial resolution of  $7.3 h^{-1}$  comoving kpc in a region of  $7.5 h^{-1}$  Mpc, accommodating the whole virial volume of the GC. The ensuing dynamic range of resolved spatial scales is sufficiently large for the emergence of turbulence.

**Galaxy cluster characteristic quantities** The galaxy cluster and its formation history are reconstructed using our implementation of a HOP halo finder<sup>35</sup> and merger history code. The virial radius is defined as the region enclosing a mass over-density  $\Delta_c = 178\Omega_m^{0.45}$  with respect to the critical density<sup>36</sup>. At redshift  $z = 0$ , the virial radius is  $R_{\text{vir}} = 1.95h^{-1}$  Mpc, and the corresponding enclosed mass,  $M_{\text{vir}} = 1.27 \times 10^{15} M_\odot$ . Also at  $z = 0$ , using  $\Delta_c = 500$  we find the characteristic radius  $R_{500} \simeq 1h^{-1}$  Mpc.

**Turbulence characterization** The characteristic quantities describing the turbulence are inferred from the analysis of the structure functions. This analysis is described in detail in<sup>3,9</sup>. Basically, we decompose the velocity into a solenoidal and a compressional component using a Hodge-Helmoltz decomposition, i.e.

$$\mathbf{v} = \mathbf{v}_s + \mathbf{v}_c, \mathbf{v}_c = -\nabla\phi, \quad \mathbf{v}_s = \nabla \times \mathbf{A}, \phi = \frac{1}{4\pi} \int \frac{\nabla \cdot \mathbf{v}}{r} d\mathbf{x}, \quad \mathbf{A} = \frac{1}{4\pi} \int \frac{\nabla \times \mathbf{v}}{r} d\mathbf{x}, \quad (4)$$

and then we compute the second and third order structure functions of velocity increments of the solenoidal component,  $\delta v_i \equiv [\mathbf{v}_s(\mathbf{x} + \mathbf{l}) - \mathbf{v}_s(\mathbf{x})]_i$ <sup>21</sup>,

$$S_i(\mathbf{l}) \equiv \langle (\delta v_i)^p \rangle, \quad (5)$$

where  $p = 2, 3$  indicates the structure function order, and  $i$  indicates the projection along or perpendicular to  $\mathbf{l}$  for the longitudinal and transverse structure functions respectively. To compute the structure functions we define sampling points randomly distributed inside the volume of interest (within  $(1/3)$  of the viral radius), and compute the velocity difference with respect to other randomly selected field points at a maximum distance of two virial radii. Once the velocity structure functions are computed, we define the velocity dispersion as the asymptotic values of the second order structure function, and the outer scale as the separation at which that asymptotic value is reached. To compute the Mach number we divide the turbulent velocity dispersions by the sound speed,  $c_s = \sqrt{\gamma P / \rho}$ , computed by evaluating the mean value of each thermodynamic quantity within the same volume in which the sampling points are collected. Finally, the turbulent dissipation rate is computed by identifying the inertial range of the second and third order structure functions of the solenoidal velocity increments (for details see ref.<sup>20</sup>).

**Code Availability** We have opted not to make the code available for practical reasons. However, the methods we adopt are published in the literature and are commonly used in the community. Amongst others, the publications mentioned in the above Methods section contain tests of our code against problems with known solutions and also with respect to solutions obtained with similar codes from independent authors.

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**Author Contributions** F.M. carried out the cosmological simulations, computed the turbulence structure functions, derived Eq. 1, 2 and 3 and wrote most of the text. A.B. analysed the structure functions, testing the self-similar nature of 2nd and 3rd order structure function within the inertial range and computing the dissipation rate. A.B. and F.M. computed the evolution of  $E_B$  and  $L_A$ .

**Competing Interests** The authors declare that they have no competing financial interests.

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**Figure 1 | High resolution simulation of a galaxy cluster in fully cosmological context.** Baryonic gas in the large scale structure of the universe (panel a; bright is high, dark is low), and around the GC centre where numerical resolution is highest (panel b; colormap inverted). Dynamic range of density (in  $cm^{-3}$ ) is  $\approx 10^6$ . The black dash-line marks the virial radius,  $R_{vir}$ , enclosing the volume that has nominally reached dynamical equilibrium. Panel c: vorticity magnitude on a scale twice the finest mesh size ( $\simeq 20$  kpc). Panel d: temperature map. Complexity is due to shocks and contact discontinuities in the turbulent flow. Dynamic range of temperature (in K) and vorticity (in  $H_0^{-1}$ ) is about  $10^3$ .

**Figure 2 | Time evolution of turbulence.** Panel a:  $\eta_{turb}$ , the ratio of ICM thermal energy contributed by turbulent dissipation,  $\int_0^t dt \rho \epsilon_{turb}$ , to the total thermal energy,  $E_{th}$ . Panel b: turbulent Mach number, the ratio of the turbulent rms velocity  $\langle (\delta u_L)^2 \rangle^{\frac{1}{2}}$  to the sound speed  $c_s$ . Panel c: volumetric turbulent dissipation rate,  $\epsilon_{turb}$  obtained in<sup>20</sup>. The error bar correspond to the variance of  $\epsilon_{turb}$ . All quantities are computed within 1/3 of the virial radius. Time in billion-yr is reported on the bottom x-axis and cosmological redshift on the top x-axis.

**Figure 3 | Time evolution of magnetic field.** Panel a:  $\beta_{plasma}$ , the ratio of ICM thermal to magnetic pressure computed as  $B^2/8\pi = C_E \int^t d\tau \rho \epsilon_{turb}(\tau)$ . Panel b:  $L_A/L$ , the ratio of Alfvén to the turbulent injection scale.  $L_A(\tau) = v_A^3/[C^{\frac{3}{2}} \langle \epsilon_{turb} \rangle]$ , where  $v_A = B/\sqrt{4\pi}$  is the Alfvén speed and  $\langle \epsilon_{turb} \rangle$  is the turbulent dissipation rate smoothed over  $\tau = 2$  Gyr with a Gaussian filter. Quantities in panels a,b refer to a volume inside 1/3 the virial radius.



The dash lines show transients to the asymptotic regime for an artificial  $t_{start} = 4.5$  Gyr. Panel c: turbulence injection scale  $L$  (solid-line) and the characteristic cluster size  $R_{500} = 0.5 R_{vir}$  (dash-line), enclosing a mass over-density of 500. Time in billion-yr (bottom x-axis) and cosmological redshift (top x-axis) are reported.

**Figure 4 | Generation and cascade of ICM hydromagnetic turbulence.** First the gravitational potential energy is converted into kinetic energy of accretion flows. These generate shear and shocks which, in addition to heat dissipation, produce fluid instabilities and baroclinic term, respectively, leading to turbulent flows. Shocks also accelerate particles through the Fermi I mechanism. Shocks do not dissipate tangential flows which will either generate turbulence, shear or shocks or a combination thereof. The turbulence cascade includes, dissipation of compressible modes at weak shocks, conversion of turbulent to magnetic energy via dynamo action, excitation of plasma waves accelerating relativistic particles through Fermi II mechanism, and of course viscous dissipation.

**Figure 5 | Spectrum of ICM hydromagnetic turbulent cascade.** Characteristic spectrum of turbulent kinetic energy in the ICM. Solid and dashed lines correspond to the solenoidal (Kolmogorov-like) and the compressional (Burgers-like) velocity field, respectively. On the X-axis, from left to right we have marked the virial scale,  $R_{vir}$ , the injection scale,  $L$ , the Ozmidov's scale,  $L_O$ , the Alfvén scale,  $L_A$ , and Kolmogorov's dissipation scale,  $\ell_{diss}$ . All quantities are time dependent and Ozmidov's scale is comparable to the injection scale, so at times turbulence in the radial direction could be suppressed by stratification.